Journal of Engineering Physics and Thermophysics, Vol. 74, No. 5, 2001

FINITE-ELEMENT PLANE MODEL OF THERMAL CONDITIONS IN SELF-PROPAGATING HIGH-TEMPERATURE SYNTHESIS OF BLANKS IN A FRIABLE SHELL

A. P. Amosov and A. F. Fedotov UDC 621.1.016.4

A two-dimensional problem of nonstationary heat conduction in a system of three bodies of finite size with an inner movable source of the first kind (combustion front) has been solved by a finite-element method. The effect of different factors on the parameters of a temperature field in self-propagating high-temperature synthesis pressing of alloys of a Ti–C–Ni system has been studied. It is found that a regime of internal cooling is realized in the synthesis products, when the temperature of the contact surface behind the combustion front remains constant and evens out over the volume of a blank. In the regime of internal cooling, the material has the highest plasticity and compactibility.

One method of obtaining high-melting compounds and materials based on them is self-propagating high-temperature synthesis (SHS), which is a kind of combustion. To obtain compact materials, the self-propagating high-temperature synthesis products heated by a combustion wave are subjected to pressing (technology of SHS pressing). As opposed to hot pressing of inert powders, when isothermal conditions are provided by external heating, in pressing the self-propagating high-temperature synthesis products are cooled constantly. Therefore, to reduce heat losses, provision should be made for thermal insulation of synthesis products from cold deforming instruments. In SHS pressing, this problem is solved by carrying out synthesis and subsequent pressing in a shell made of friable material (sand). The proclivity of the products of synthesis to plastic deformability and compactibility are first of all conditioned by the temperature mode of deformation. In this connection, of scientific and practical interest are the studies of a thermal regime in cooling combustion products in the heat-insulating shell.

The well-known works dealing with the study of thermal conditions of SHS pressing consider an axisymmetric problem of nonstationary heat transfer with combustion-front motion along the symmetry axis of the cylindrical system [1]. In SHS pressing, a blank having the shape of a round or a square plate is placed into a cylindrical press die perpendicularly to the symmetry axis. The combustion reaction is initiated from the side surface or from the center of the charge blank, and the combustion front moves perpendicularly to the symmetry axis. In this form, the problem of a thermal regime has been not studied.

The paper is aimed at mathematical simulation and derivation of the laws that govern the formation of a thermal regime in combustion of exothermal mixtures in a heat-insulating shell. A finite-element method is used to simulate the process of heat exchange of the synthesis products with the ambient medium.

Self-propagating high-temperature synthesis in a charge blank is initiated locally (at a point) and then the process proceeds in the mode of layer-by-layer combustion. The thermokinetic parameters are the temperature T_c and rate u_c of combustion, which are assumed to be known. The blank is placed in a sand shell and a steel instrument. As a result, the object of simulation is a three-element system with regions of finite

---

Samara State Technical University, Samara, Russia; email: mvm@dp.sstu.samara.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 5, pp. 160–166, September–October, 2001. Original article submitted December 18, 2000; revision submitted February 23, 2001.

 1062-0125/01/7405-1272$25.00

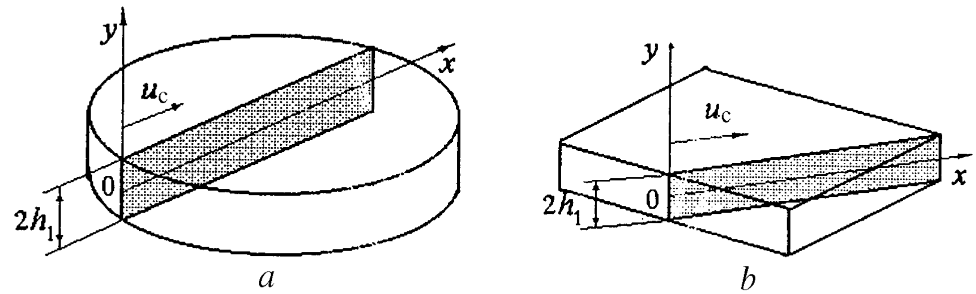


Fig. 1. Schematic diagram of the cutting out of a plane layer from round (a) and square (b) blanks.

dimensions with a moving internal boundary of the first kind (combustion front). There is nonstationary heat exchange between the elements of the system.

The parameters of the thermal regime and the dimensionality of the problem being solved depend on the coordinates of the point at which the combustion reaction is initiated. On ignition from the side surface, irrespective of the shape of the plate, heat is exchanged in the direction of three spatial coordinates. When the reaction is initiated from the center of the square plate up to the moment of the combustion-wave egress to the side face, heat exchange is first two-dimensional (axisymmetric) and then three-dimensional. In the round plate, there is axisymmetric heat exchange during the whole period of combustion. Technologically, it is easier to ignite the charge blank from its side surface, and this method has found the widest practical application. From a mathematical point of view, solution of a three-dimensional problem of nonstationary heat conduction in a multilayer system is cumbersome and laborious. To estimate the compactibility of a temperature-nonuniform blank one needs only the data on the temperature of the coldest regions. This information can be obtained in considering the two-dimensional model of a burning plane layer of unit thickness with thermally insulated side surfaces. The layer is cut so that its length corresponds to a maximum possible way of combustion. Accordingly, the sectional plane passes through the ignition point and the point at the greatest distance from it. This model allows description of the temperature field in the coldest (onset of combustion) and hottest (end of combustion) zones of the blank. When the round blank is ignited from its side, the cross section follows its diameter; in the square blank, the cross section passes from the center of the side face, then through the center of the side face and the opposite vertex of the square (Fig. 1). The farther the side surfaces of the blank from the burning layer, the more correct the model adopted. We note that for the plane model the parameters of the computational relations and the results obtained will be invariant to the shape of the plate.

The accuracy of pressure moulding depends on the contact rigidity of a deforming device. In pressing in a friable shell, forming and compaction occur owing to the setting of the blank, with its radial dimensions remaining virtually unchanged. Therefore, in SHS pressing, the rigidity (thickness) of the shell between the blank and the die or the bottom of the counter die is most important. Of secondary importance are the radial dimensions of the shell between the blank and the counter-die wall.

In view of the above, the model adopted considers heat exchange between the blank and the die (or the counter-die bottom), with the shell thickness between them being varied. The shell thickness in the radial direction is selected taking into account the condition of the absence of through heating of the shell and heat removal by the counter-die walls. The calculations showed that for typical time parameters of SHS pressing complete thermal insulation of the blank from the side walls of the counter die is reached at a shell thickness of l_{sh} = 12 mm.

Figure 2 shows a computational scheme for a three-layer system consisting of a plane layer of combustion products 1 of length l, heat-insulating shell 2, and die 3 of diameter $D = l + 2l_{sh}$ mm. By virtue of the axial symmetry, we consider half the height of the blank. Combustion of the blank begins from the left

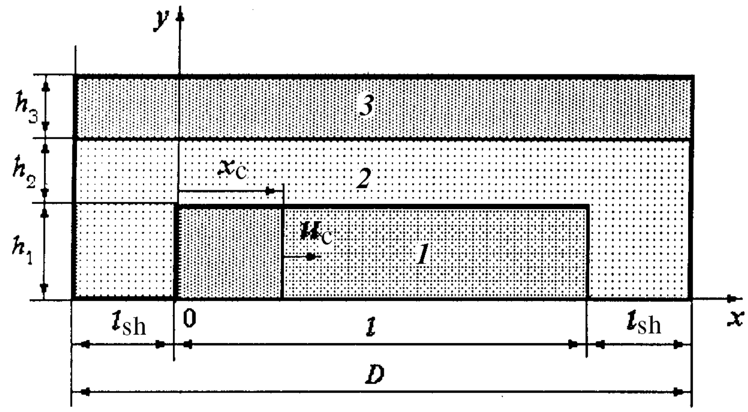


Fig. 2. Schematic diagram of the object of simulation: 1) blank; 2) shell; 3) die.

plane (x_0 = 0); the combustion front is taken to be plane and moving toward the x axis at a constant velocity u_c.

A mathematical model of heat exchange in SHS pressing includes:

(1) a system of three differential equations of nonstationary heat conduction in the Cartesian coordinates:

$$c_i \rho_i \frac{\partial T_i(x, y, t)}{\partial t} = \lambda_i \left(\frac{\partial^2 T_i(x, y, t)}{\partial x^2} + \frac{\partial^2 T_i(x, y, t)}{\partial y^2} \right); \quad (1)$$

(2) boundary conditions: the fourth-order conditions at the blank–shell y_{1-2} and the shell–device y_{2-3} boundaries:

$$\lambda_1 \frac{\partial T_1(x, y_{1-2}, t)}{\partial y} = \lambda_2 \frac{\partial T_2(x, y_{1-2}, t)}{\partial y}; \quad T_1(x, y_{1-2}, t) = T_2(x, y_{1-2}, t);$$

$$\lambda_1 \frac{\partial T_2(x, y_{2-3}, t)}{\partial y} = \lambda_2 \frac{\partial T_3(x, y_{2-3}, t)}{\partial y}; \quad T_2(x, y_{2-3}, t) = T_3(x, y_{2-3}, t); \quad (2)$$

the third-order conditions at the device–ambient medium boundary ($y_3 = h_1 + h_2 + h_3$):

$$\lambda_3 \frac{\partial T_3(x, y_3, t)}{\partial n} + \alpha (T_3 - T_s) = 0; \quad (3)$$

(3) initial conditions:

$$T_1(0, y_1, 0) = T_c; \quad T_2(x, y, 0) = T_s; \quad T_3(x, y, 0) = T_s; \quad (4)$$

(4) equation of combustion-front motion:

$$x_c = u_c t;$$

(5) the temperature of a moving boundary of the first kind (combustion front):

$$T(x_c, y_1, t) = T_c ;$$

(6) condition of symmetry of a temperature field relative to the y axis:

$$\frac{\partial T_1(x, 0, t)}{\partial y} = 0 ;$$

(7) conditions of adiabaticity ahead of the combustion front and on the outer side surfaces of the shell:

$$\frac{\partial T_2(-l_{sh}, y, t)}{\partial x} = 0 ; \quad \frac{\partial T_2(l + l_{sh}, y, t)}{\partial x} = 0 ; \quad \frac{\partial T_1(x_c - 0, y_1, t)}{\partial x} = 0 .$$

Solution of Eqs. (1) with boundary conditions (2) and (3), and initial conditions (4) is equivalent to finding a minimum of the following variational functional:

$$J = \sum_{i=1}^{3} \frac{1}{2} \int_{V_i} \lambda_i \left[\left(\frac{\partial T_i}{\partial x} \right)^2 + \left(\frac{\partial T_i}{\partial y} \right)^2 + 2 c_i \rho_i \frac{\partial T_i}{\partial t} T_i \right] dV_i + \int_{S_3} \frac{\alpha}{2} (T_3 - T_s)^2 \, dS_3 . \quad (5)$$

A distinguishing feature of functional (5) is that at the stage of combustion the volume V_1 of hot synthesis products that participate in heat exchange is the time function

$$V_1 = h_1 u_c t .$$

The unknown temperature field was found by a finite-element method. When breaking into finite elements, the entire domain is first overlaid with a rectangular grid and then the rectangles obtained are divided by diagonals into two triangles. In the regions of the boundaries of contact heat exchange of the blank with the shell with high temperature gradients, the grid of finite elements is made denser.

The condition of stationarity of functional (5) leads to the following discrete differential equation in a matrix form:

$$[C] \frac{\partial \{T\}}{\partial t} + [\Lambda] \cdot \{T\} = \{F\} . \quad (6)$$

Matrix differential equation (6) was solved by a method of finite-differences using an implicit difference scheme

$$([C] + \Delta t \cdot [\Lambda]) \cdot \{T_k\} = [C] \cdot \{T_{k-1}\} + \Delta t \cdot \{F\} . \quad (7)$$

The elements of the matrices $[C]$, $[\Lambda]$, and $\{F\}$ are determined from the known relations for plane triangular elements [2].

The technological time of the formation of a temperature field t consists of combustion time t_c and the time of delay in pressing t_d: $t = t_c + t_d$. The magnitude of the time pause t_d is composed of the time of operation of the executive system of the press (≈ 0.5 sec) and the time set by a researcher. Motion of the combustion front was simulated by a step-by-step increase in the number of finite elements of the blank

TABLE 1. Thermophysical Properties of Materials

Material	λ, W/(m·K)	c, J/(kg·K)	ρ, kg/m^3	T_c, ^{o}C	u_c, mm/sec
TiC–20% Ni	12.1	967.5	2700	2400	15
TiC–28% Ni	12.4	955.4	2810	2240	10
TiC–35% Ni	12.7	945.5	2910	2080	6
Sand [6]	0.326	795.0	1500	–	–
Die (steel) [5]	32.0	561.0	7800	–	–

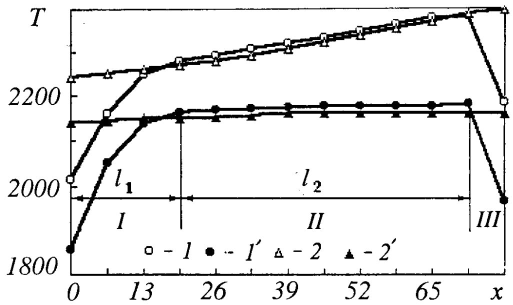
which participate in heat exchange. With a rectangular finite-element grid, the whole region is a set of vertical columns and horizontal layers. In the approximation of a plane combustion front, the volume of the synthesized material increased for one step by the volume of elements of one column of the blank. At the first step, only the elements of the first column of the blank of width Δx_1 participate in heat exchange. At the second step, the combustion front moves by the width of the elements of the second column of the blank Δx_2; at the third step, it moves by the width of the elements of the third column of the blank Δx_3, and so on. At the nth step, the combustion time t_{cn} of a new column of the blank of width Δx_n is

$$t_{cn} = \frac{\Delta x_n}{u_c} \,. \tag{8}$$

During this period the blank also starts to cool. To determine optimum steps along the time axis, the solution of one-dimensional equation (7) was compared with the exact analytical solution of the one-dimensional problem of cooling an infinite layer of substance in an unbounded medium at the boundary conditions of fourth kind [3]. In analyzing the solutions it was found that the value of the time step t_{cn} calculated from Eq. (8) and being equal to about 0.5–1 sec for real values of Δx_n and u_c is a rather rough discretization of the time axis. Therefore, the time t_{cn} was divided into m intervals, and Eq. (7) was solved with the step $\Delta t_n = t_{cn}/m$. It is found that at $m = 5$ the numerical solution differs from the analytical one by no more than 1%. The system of linear equations (7) was solved by the Zeidel iteration method with an accuracy of 0.5^{o}C.

The laws governing the formation of a temperature field in a plane combustion layer were studied in SHS in a Ti–C–Ni system. Combustion of this system occurs owing to a highly exothermal reaction of the formation of titanium carbide: Ti + C = TiC; inert nickel serves as a binder. The thermophysical properties of SHS products with different contents of nickel, sand, and a steel die are presented in Table 1. The data on the combustion temperature T_c are taken from [4]; the value of the combustion rate u_c is determined experimentally. The combustion temperature T_c for all the compositions exceeds the temperature of the eutectics of the TiC–Ni system, coming to 1280^{o}C, and the synthesis products consist of solid particles of titanium carbide and carbide–nickel melt. The thermophysical properties of porous solid–liquid material were calculated by the relationships of [6] and, in accordance with the diagram of state, the amount of liquid and solid phases, the composition of the melt, and porosity of the material equal to 50% were taken into account. The heat-transfer coefficient was taken equal to $\alpha = 44$ W/(m^2·K).

In the first stage, we studied a thermal regime in synthesis of the blanks of standard nomenclature and at regular technological parameters of the process. We considered combustion of a layer of length $l = 78$ mm; this corresponds to synthesis of a square blank with a side of 70 mm with ignition at the center of the side face. The height of the blank and of the layer was $2h_1 = 14$ mm, the shell thickness was $h_2 = 10$ mm, the die thickness was $h_3 = 15$ mm, and the counter-die had a diameter $D = 102$ mm. Figure 3 shows the distribution of temperature along the layer length after synthesis of the TiC–20% Ni alloy and the time of pressing delay $t_d = 0.5$ sec. The character of the change of temperature on the contact surface T_{cont} and at the center T_{cent} shows that a nonuniform temperature field is formed in the layer. This nonuniformity is due to two factors. First, the plane layer of finite length has three boundaries of contact heat exchange, which serve



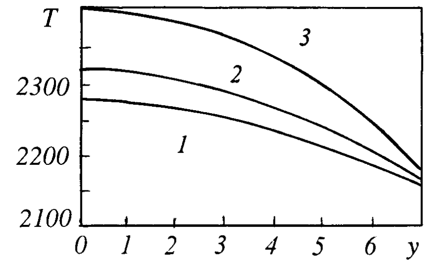


Fig. 3. Change in the temperatures of the contact surface T_{cont} (1′, 2′) and the center T_{cent} (1, 2) along the layer length: 1 and 1′) solution of a two-dimentional problem by a finite-element method; 2 and 2′) analytical solution of a one-dimensional problem. T, ^{o}C; x, mm.

Fig. 4. Temperature distribution over the layer thickness: 1) x = 22 mm; 2) 30; 3) 72.

as heat sinks: the bearing and two side planes. The "coldest" zones with a high temperature gradient are formed close to these boundaries. Second, in heating of a blank by a moving front, the time of cooling t_{cool} of separate zones depends on their position (the coordinate x) relative to the ignition point:

$$t_{cool} = \frac{l - x}{u_c} + t_d , \quad 0 \le x \le l . \tag{9}$$

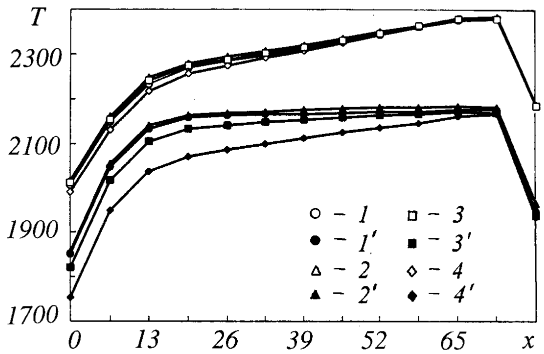
Away from the ignition plane and on increase in the coordinate x, the cooling time decreases and the temperature of the blank increases.

On complete combustion of the layer, three characteristic temperature zones related to the corresponding boundary of contact heat exchange are formed in the layer (Fig. 3). Zone I, for which the largest time of cooling is typical, is in the vicinity of the ignition plane; this zone is the coldest. On moving away from the ignition plane, the effect of this boundary as a heat sink decreases and zone II is formed; it is characterized by a weak dependence of the contact temperature T_{cont} on the time of cooling and the coordinate x along the whole zone $T_{cont} \approx$ const. The value of T_{cont} in zone II is approximately equal to the temperature T_{cont0}, which is instantly set at the contact boundary of an infinite layer placed into an infinite medium [3]:

$$T_{cont0} = T_s + (T_c - T_s) \frac{K_{\varepsilon 1}}{K_{\varepsilon 1} + K_{\varepsilon 2}} . \tag{10}$$

The temperature at the center of the layer T_{cent} (at y = 0) increases with distance from the ignition plane and decrease in the time of cooling. Zone III forms near the finite side plane ($x = l$). This zone is characterized by a sharp decrease in the temperatures T_{cont} and T_{cent} with approach to the side boundary of contact heat exchange, on which combustion stopped.

Figure 4 shows the distribution of temperature over the layer thickness (the coordinate y) in sections at the boundary of zones I and II (x = 22 mm, t_{cool} = 4.2 sec), at the center of the blank (x = 39 mm, t_{cool} = 3.1 sec), and at the boundary of zones II and III (x = 7.2 mm, t_{cool} = 0.9 sec). The sharp difference between the temperatures of the central and contact volumes is typical of a small time of cooling (curve 3). Inside zone II, with increase in the time of cooling at a virtually constant value of the temperature T_{cont} (for y = 7 mm), the temperature levels out over the blank thickness owing to cooling of the central volumes. Thus, a mode of internal cooling is realized in zone II [3].



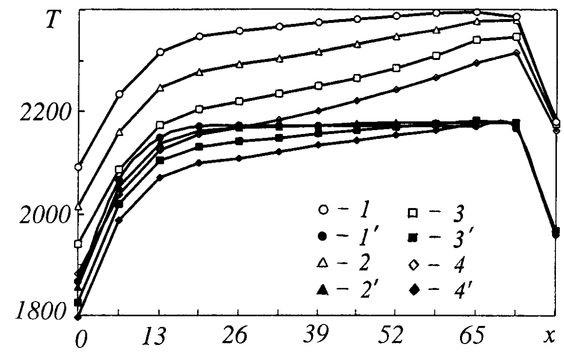


Fig. 5. Influence of the shell thickness h_2 on the distribution of temperature T_{cent} (1–4) and T_{cont}(1′–4′) along the layer length at $2h_1$ = 14 mm and t_d = 0.5 sec: 1 and 1′) h_2 = 10 mm; 2 and 2′) 2; 3 and 3′) 1.5; 4 and 4′) 1.0.

Fig. 6. Influence of the thickness of the blank $2h_1$ on the distribution of temperature T_{cent} (1–4) and T_{cont} (1′–4′) along the layer length at h_2 = 10 mm and t_d = 0.5 sec: 1 and 1′) $2h_1$ = 20 mm; 2 and 2′) 16; 3 and 3′) 10; 4 and 4′) 8.

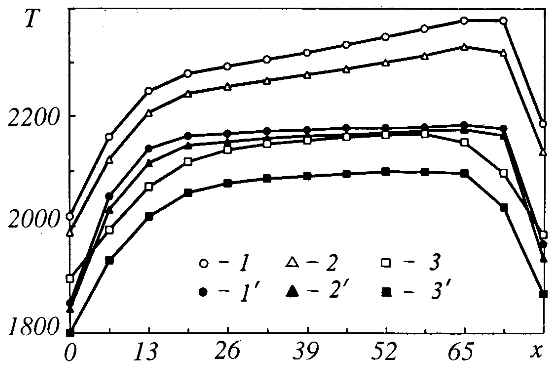
The results of numerical calculation were compared with the results of the analytical solution of a one-dimensional problem of cooling an infinite layer placed into an infinite medium [3]. The time of cooling the infinite layer was taken equal to the time of cooling the section with the variable coordinate x in combustion of the layer of finite length and was calculated from relation (9). The results of the analytical solution and of the solution by the finite-element method (Fig. 3) are virtually the same (the difference does not exceed 1%). From this it follows that a temperature field in zone II with internal cooling is formed as a result of one-dimensional contact heat exchange on the bearing plane.

We consider the effect of different factors on a thermal regime of SHS pressing of the products of synthesis of the Ti–C–20% Ni system.

Of primary importance for the thermal regime is the thickness of the shell between the blank and the device. The results of calculation show that reduction of the shell thickness from h_2 = 10 mm (recommendations of the standard technology) to h_2 = 2 mm virtually does not exert a substantial effect on the temperature field (Fig. 5). When $h_2 < 2$ mm, the process of blank cooling is enhanced, with the contact volumes of the blank being cooled to a greater extent than the central ones. Correspondingly, the length of zone II with internal cooling decreases, and for $h_2 \le 1$ mm this zone is absent.

The effect of the blank thickness $2h_1$ on the laws governing a change in the temperatures T_{cont} and T_{cent} is shown in Fig. 6. Thin blanks with $2h_1 \le 8$ mm are cooled faster, and zone II with a mode of internal cooling does not succeed in forming inside them. On increasing $2h_1$ from 8 to 16 mm, the cooling rate decreases; the temperature T_{cont} rises, and zone II becomes longer. When $2h_1 > 16$ mm, the temperature T_{cont} over the entire length of the layer and the dimensions of the characteristic temperature zones remain unchanged.

The time of delay in pressing t_d is an important technological parameter. As t_d increases the layer cools, and the temperature gradients decrease gradually along the length and height of the layer (Fig. 7). When $t_d \le 2$ sec, a regime of internal cooling is realized when the temperature of the contact surface T_{cont} remains practically unchanged. At the same time, the inner volumes of the blank with a temperature T_{cent} cool noticeably. The boundary between zones II and III, which at the time of complete combustion of the blank was the hottest, cools more rapidly. As t_d increases owing to heat conduction, the temperature T_{cent} levels along the layer length, and when $t_d \ge 10$ sec, this high-temperature region disappears.



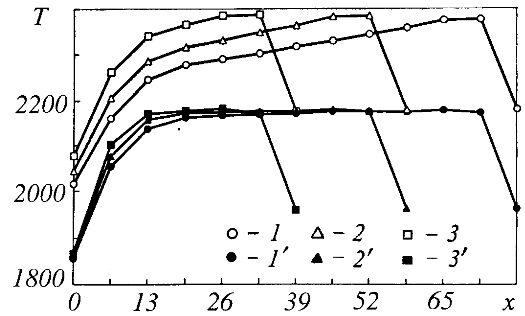


Fig. 7. Influence of time of delay in pressing t_d on the distribution of temperature T_{cent} (1–3) and T_{cont} (1′–3′) along the layer length at $2h_1$ = 14 mm and h_2 = 10 mm: 1 and 1′) t_d = 0.5 sec; 2 and 2′) 2; 3 and 3′) 10.

Fig. 8. Influence of the length of the plane layer l on the distribution of temperature T_{cent} (1–3) and T_{cont} (1′–3′) along the layer length at $2h_1$ = 14 mm, h_2 = 10 mm, and t_d = 0.5 sec: 1 and 1′) l = 78 mm; 2 and 2′) 58.5; 3 and 3′) 39.

TABLE 2. Limiting Dimensions of the Shell and Temperature Zones of the Blank for the Ti–C–Ni System

Composition of a mixture	T_{cont0}, °C	Time $t_c + t_d$, sec	$2h_1^*$, mm	h_2^*, mm	l_{1min}, mm	l_{2max}, mm
Ti–C–20% Ni	2162	5.7	16	2.0	22	51
Ti–C–28% Ni	2018	8.3	18	3.5	26	46
Ti–C–35% Ni	1874	13.5	22	4.0	32	39

Figure 8 shows a change in the temperatures T_{cont} and T_{cent} in the layer with variation of its length. Irrespective of the length l, the three characteristic zones considered above form in the layer. The temperature T_{cont} in the respective zone has virtually the same values. The difference in the time of combustion and cooling of layers of different lengths manifests itself in the level of the temperature T_{cent}: as the radial dimensions of the blank and the combustion length l decrease, the cooling time (9) decreases, and a higher temperature T_{cent} remains inside the blank.

The ability of a nonuniformly heated body to undergo plastic deformation is determined by the temperature of the coldest zones. In our case, this is the temperature of the contact surface T_{cont}. A maximum value of the contact temperature $T_{cont\,max} \approx T_{cont0}$ gives an upper estimate of the temperature of deformation of synthesis products. The region of the regime of internal cooling with the temperature $T_{cont\,max}$ determines the volume of the material with the largest plasticity and compactibility. The results of computational experiments (Fig. 5 and 6) show that as the thicknesses of the blank $2h_1$ and of the shell h_2 increase, the length of the zone of one-dimensional heat exchange l_2 increases, and at certain values of $2h_1^*$ and h_2^* it takes its limiting value l_{2max}. Accordingly, zone I of two-dimensional heat exchange has limiting minimum dimensions l_{1min}. Table 2 presents the results of calculations of the limiting dimensions of the blank $2h_1^*$ and of the shell h_2^* in SHS pressing of the products of synthesis of the Ti–C–Ni system (for all the variants of l = 78 mm, t_d = 0.5 sec). According to (9), as the content of nickel increases and the rate of combustion u_c decreases (Table 1), the time of cooling t_{cool} of synthesis products and the time of warming-through of the shell increase. This results in an increase in the limiting thicknesses of the blank $2h_1^*$ and of the shell $2h_{2*}^*$ at which the zone with the regime of internal cooling is the longest l_{2max}. With an increase in the content of Ni, the parameter l_{2max} and the volume of the material with maximum plasticity and compactibility decrease.

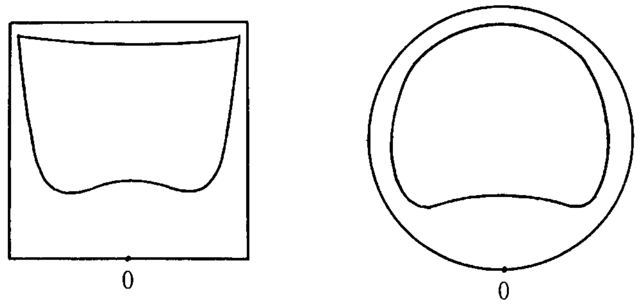


Fig. 9. Isotherm $T_{\text{cont}} = T_{\text{cont max}}$ for square and round blanks (0, initiation point).

A model of a plane layer describes a thermal regime in the cross section of a blank along the trajectory of any point of the combustion front. From the set of solutions for the trajectories of several points of the combustion front it is possible to obtain an estimate of the thermal regime of the whole blank. Using this method, we constructed the isotherm $T_{\text{cont}} = T_{\text{cont max}}$ for a square plate with a side 70 mm and a round plate with a diameter of 78 mm in SHS pressing of the TiC–20% Ni alloy (Fig. 9). The thickness of the blank was $2h_1 = 16$ mm, the thickness of the shell was $h_2 = 2$ mm, and the time $t_{\text{d}} = 0.5$ sec. After pressing, the region inside the isotherm with the regime of internal cooling will have a maximum density of the material. The values of the maximum length and time of combustion are the same for both variants of the plate shape. At the same time, a volume of the material with $T_{\text{cont}} = T_{\text{cont max}}$ for the round plate is larger than for the square one.

Thus, the two-dimensional model of heat exchange of a burning plane layer made it possible to reveal the main laws governing the formation of a thermal regime in combustion of blanks of finite dimensions in a heat-insulating sand shell. The considerable nonuniformity caused by the presence of the boundaries of contact heat exchange and nonisochronicity of the heating of a blank by a moving combustion front is typical of the temperature field in synthesis products. In nonstationary heat exchange, in the blank–shell–device system, the existence of a regime of internal cooling is possible, in which the temperature of the contact surface behind the combustion front remains constant and levels over the blank volume. The size of the region with the regime of internal cooling depends on the thermokinetic parameters of SHS mixtures, on the shape, and on the dimensions of a synthesized blank and a heat-insulating shell. In the regime of internal cooling, material has the greatest plasticity and compactibility. Correspondingly, the optimal technological parameters of the process can be determined from the condition of a maximum of the volume of a blank with a regime of internal cooling.

NOTATION

T_i, temperature of bodies; c_i, ρ_i, and λ_i, specific heat, specific density, and thermal conductivity of the bodies of the system; h_i, thickness of the bodies of the system; V_i, volume of the bodies of the system; i, index of a body in the system: 1) blank, 2) shell, 3) die; t, time; x, y, Cartesian coordinates; α, heat-transfer coefficient; T_{s}, temperature of a medium; n, normal to the boundary surface; S_3, area of the die with convective heat exchange; T_{c}, combustion temperature; u_{c}, combustion rate; $[C]$, matrix of heat capacity; $[\Lambda]$, matrix of thermal conductivity; $\{F\}$, vector of heat loads; $\{T_{k-1}\}$ and $\{T_k\}$ matrices of nodal values of temperature at the beginning and end of the time interval Δt; x_{c}, current coordinate of the combustion front; l, layer length; l_{sh}, radial thickness of the shell; D, diameter of the press die; t_{c}, time of combustion; t_{d}, delay time of pressing; t_{cool}, cooling time; T_{cont}, contact temperature; T_{cont0}, initial contact temperature; T_{cent}, temperature at the center of the blank; $K_{\varepsilon 1} = \sqrt{\lambda_1 c_1 \rho_1}$ and $K_{\varepsilon 2} = \sqrt{\lambda_2 c_2 \rho_2}$, criteria of thermal activity of the blank and the shell. Subscripts: sh, shell; c, combustion; d, delay; cont, contact; cent, center; s, medium.

REFERENCES

1. L. S. Stel'makh, N. N. Zhilyaeva, and A. M. Stolin, *Inzh.-Fiz. Zh.*, **63**, No. 5, 623–629 (1992).
2. L. J. Segerlind, *Applied Finite Element Analysis* [Russian translation], Moscow (1979).
3. A. V. Luikov, *Theory of Heat Conduction* [in Russian], Moscow (1967).
4. A. S. Rogachev, V. M. Shkiro, I. D. Chausskaya, and M. V. Shvetsov, *Fiz. Goreniya Vzryva*, **24**, No. 6, 86–93 (1988).
5. A. I. Pekhovich and V. M. Zhidkikh, *Calculation of a Thermal Regime of Solids* [in Russian], Leningrad (1976).
6. G. N. Dul'nev and Yu. P. Zarichnyak, *Thermal Conductivity of Mixtures and Composite Materials* [in Russian], Leningrad (1974).